Indian Statistical Institute
Bangalore Centre
B.Math (Hons.) III Year 2011-2012
Second Semester
Sample Surveys and Design of Experiments
Mid-semester Examination Date : 29.02.12

Answer as many questions as possible. The maximum you can score is 60.
All the notation have their usual meaning. State clearly the results you use.

1. Consider a simple random sample (SRS) of size n from a population of size N, collected **with replacement (WR)**.

   (a) Show that the sample mean is unbiased for population mean.

   (b) Obtain the expression for the variance of the sample mean.

   (c) A sampler decided to take a stratified random sample **without replacement (WOR)** from every stratum, using proportional allocation. Write down the expression for the variance of the sample mean (proof is not required). Show that stratification leads to gain in the precision (over simple random sample) if within strata variability is small.

   [3 + 6 + 6 = 15]

2. A sample survey was to be taken up to collect information on a variable Y from a population of N units. Information on an auxiliary variable X was available for every unit. It was decided to use a sampling scheme in which units were to be selected with replacement such that the probability (P_i) of selecting the ith unit was proportional to X_i.

   (a) Explain Lahiri's method of selection. Prove that it indeed selects the ith unit with probability P_i.

   (b) Consider the estimator $\bar{z} = (1/n)\sum_{i=1}^{n} z_i$, where $z_i = y_i/P_i$. Prove that $\bar{z}$ is unbiased for the population total and show that its variance is

   $$(1/n) \sum_{i \neq j,\ i,j=1}^{n} P_i P_j (z_i - z_j)^2.$$

   (c) What condition the relationship between the variables X and Y must satisfy so that the present scheme is 'better' than SRSWR ?

   [(2+5)+ (2+5) + 5 = 19]

3. When is a block design said to be orthogonal ? Show that a block design is orthogonal if and only if

   $$n_{ij} = r_i k_j / n.$$

   [2+8=10]

4. (a) Define a balanced incomplete block design [BIBD]. Show that the following relations hold among the parameters of a $BIBD$.

$$\begin{aligned} bk &= vr \\ \lambda(v-1) &= r(k-1) \\ \text{and } b &\geq v. \end{aligned}$$

(b) Find the variance of the BLUE of a treatment contrast $l'\tau$ obtained from a BIBD.

(c) Show that every pair of blocks of an SBIBD have the same number of treatments in common.

[(2+5) + 3 + 4 = 14]

5. When are two latin squares are said to be mutually orthogonal ? Suppose s is a prime or a prime power. Show that there are $s-1$ mutually orthogonal latin squares of order s,

(c) If s is a prime or a prime power, show that a BIBD with $v = s^2, k = s, \lambda = 1$ exists. Illustrate with $s = 4$.

[(2+5) + 7 = 14]